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Curing the infinite ghost tower in 4d Siegel superparticle

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Abstract: On an explicit example of the Siegel superparticle we study an alternative to the harmonic superspace approach. The latter seems to be the only method for quantizing infinitely reducible first class constraints currently available. In an appropriately extended phase space, the infinite ghost tower is effectively canceled by that coming from the sector of auxiliary variables. After a proper BRST treatment the theory proves to be of rank two which correlates well with the results obtained earlier within the framework of the harmonic superspace approach. The advantage of the novel technique, however, is the existence of an explicit lagrangian formulation and the standard spin-statistics relations which hold for all the variables involved.

Keywords: BRST Quantization, Superstrings and Heterotic Strings, Superspaces.

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Contents

1. Introduction

The superparticle due to Siegel [1] (later referred to as AB superparticle) has originally been proposed as a way to avoid problematic second class constraints intrinsic to a conventional superparticle (superstring) theory without sacrificing manifest Lorentz covariance. To compensate a mismatch [2] in the number of degrees of freedom between the AB model and the conventional formulation [3], it was suggested [4] to introduce further (C and D) constraints, the total set forming a closed algebra. The equivalence of the $ABCD$ theory to the conventional superparticle (superstring) has been claimed in ref. [5], thus suggesting an intriguing alternative to the standard formalism.[1] Yet, although in the modified theories constraints

[1]It has to be mentioned, however, that only in the string case the proof seems to be completely consistent. For the mechanics analogue the higher order fermionic C constraints were treated differently (*a la* Gupta-Bleuler) from others.

do form a closed algebra and are straightforward to be realized quantum mechanically (operator quantization) [6], the results of the path integral quantization [7]–[11] seem to be intractable because of infinite reducibility of the fermionic constraints involved.

A common way to attack the latter problem, which proved to be successful for the original superparticle and superstring [12, 13, 14] (field theory applications were proposed earlier in ref. [15]), is to make use of Lorentz harmonics to extract linearly independent components from the fermionic constraints in a covariant fashion. For the AB model this has been accomplished in ref. [16] yielding a theory of rank two after a proper BRST treatment. It has to be noted, however, that the approach of ref. [16] is essentially hamiltonian. Moreover, the standard spin-statistics relations do not hold for some of the variables involved. Another serious problem is the noncompactness of the coset space parametrized by the harmonics used (see the discussion in ref. [13]).

Recently, an alternative technique to cure the infinite ghost tower problem intrinsic to the Siegel superparticle, superstring has been proposed in ref. [17]. The idea was to appropriately extend the original phase space and then effectively cancel the infinite ghost tower by that coming from the sector of auxiliary variables.

In the present paper we investigate in full details this, looking somewhat exotic, possibility and show that the result of the quantization correlates well with that obtained previously within the framework of the harmonic superspace approach [16]. The advantage of the novel scheme, however, is the presence of an explicit lagrangian formulation and the standard spin-statistic relations which hold for all the variables. For simplicity of the presentation we restrict ourselves to the four dimensional case. With some modifications, however, this can be generalized to other dimensions [17].

In the next section we review the AB model in $d = 4$. It is shown that, in contrast to the common opinion, a ghost free and unitary quantum mechanics can be constructed if one sacrifices conventional conjugation properties for fermionic operators and chooses a specific modification. A light-cone Hilbert space is explicitly constructed which can be identified with the one particle sector of the quantized supersymmetric massless Wess-Zumino model. In section 3, following the ideology of an earlier work [17], we embed the original Siegel model into an appropriately extended configuration space. In the hamiltonian framework, the variables from the auxiliary sector turn out to be subjected to *reducible* constraints like those entering Siegel's theory. These can further be used to put the complete constraint set into the form of *first* stage of reducibility, the latter admitting straightforward path integral quantization [18]. Section 4 contains the construction of the BRST charge in the minimal ghost sector. The theory in the extended phase space proves to be rank two. This is in perfect agreement with the analysis of the alternative harmonic superspace approach. An extension to the non-minimal ghost sector and the path integral quantization are accomplished in section 5. We conclude with some remarks on possible further developments of the formalism in section 6. Appendix A contains the light-cone notation and technical points related to the hamiltonian analysis of section 3. Throughout the paper we use the spinor notation from ref. [19].

2. Review of the 4d AB model. No negative norm states and unitarity in the physical subspace.

Retaining only first class constraints of the conventional superparticle[2] (see ref. [6] for the details of the Dirac procedure)

$$p^2 = 0\,, \qquad (p_\theta \sigma^n p_n)_{\dot\alpha} = 0\,, \qquad (\sigma^n p_{\bar\theta} p_n)_\alpha = 0\,, \tag{2.1}$$

where $(p_n, p_{\theta\alpha}, p_{\bar\theta\dot\alpha})$ are momenta conjugate to the configuration space variables (x^n, $\theta^\alpha, \bar\theta^{\dot\alpha}$), one discovers the constraint set to describe Siegel's superparticle [1]. The corresponding canonical hamiltonian reads

$$H = \frac{1}{2} p^2\,. \tag{2.2}$$

Owing to the null vector p_n entering the problem, only half of the fermionic constraints is linearly independent. In particular, the identity

$$(p_\theta \sigma^n p_n)_{\dot\alpha} Z_1{}^{\dot\alpha\alpha} + Z_1{}^\alpha p^2 \equiv 0\,, \tag{2.3}$$

where $Z_1{}^{\dot\alpha\alpha} = (\tilde\sigma^n p_n)^{\dot\alpha\alpha}$, $Z_1{}^\beta = p_\theta{}^\beta$, is satisfied. On the constraint surface not all of the functions $Z_1{}^{\dot\alpha\alpha}$ prove to be independent

$$Z_1{}^{\dot\alpha\alpha} Z_{2\alpha\dot\beta} \approx 0\,, \qquad Z_{2\alpha\dot\beta} = (\sigma^n p_n)_{\alpha\dot\beta}\,. \tag{2.4}$$

Apparently, this process can be continued, the system at hand being infinite stage of reducibility following the terminology of ref. [18].

Proceeding to the light-cone analysis of the model, one imposes the conventional gauge in the fermionic sector ($A^\pm = \pm\frac{1}{\sqrt2}(A^0 \pm A^3)$)

$$\theta\sigma^+ = 0\,, \qquad \sigma^+\bar\theta = 0\,, \tag{2.5}$$

[2]Conventions adopted in this section are $(\theta^\alpha)^* = \bar\theta^{\dot\alpha}$, $(p_{\theta\alpha})^* = -p_{\bar\theta\dot\alpha}$.

or, reducing this to components

$$\theta^2 = 0\,, \qquad \dot{\bar{\theta}}^2 = 0\,. \tag{2.6}$$

The partially reduced phase space includes then three pairs[3] (x^n, p_n), (θ, p_θ), $(\bar\theta, p_{\bar\theta})$, these obeying usual canonical commutation relations and the conjugation properties (x^n, p_n are real)

$$(\theta)^* = \bar\theta\,, \qquad (p_\theta)^* = -p_{\bar\theta}\,. \tag{2.7}$$

The gauge fixed hamiltonian action acquires the form

$$S = \int d\tau \left\{ p_m \dot{x}^m + p_\theta \dot\theta + p_{\bar\theta}\dot{\bar\theta} - \frac{1}{2}p^2 \right\}. \tag{2.8}$$

Going over to the quantum description ($\{\hat\theta, \hat{p_\theta}\} = i, \{\hat{\bar\theta}, \hat{p_{\bar\theta}}\} = i$), it is customary to require a scalar product in a Hilbert space to respect the conjugation property (2.7), i.e.

$$\hat\theta^+ = \hat{\bar\theta}\,, \qquad \hat{p_\theta}^+ = -\hat{p_{\bar\theta}}\,. \tag{2.9}$$

This, however, immediately leads us to the conclusion that there are negative norm states in such a quantum space. Actually, introducing the operators

$$\begin{aligned}
\hat a &= \frac{1}{\sqrt2}\left(\hat\theta - i\hat p_{\bar\theta}\right), \qquad & \hat a^+ &= \frac{1}{\sqrt2}\left(\hat{\bar\theta} - i\hat p_\theta\right), \qquad & \left\{\hat a, \hat a^+\right\} &= 1\,,\\
\hat b &= \frac{1}{\sqrt2}\left(\hat\theta + i\hat p_{\bar\theta}\right), \qquad & \hat b^+ &= \frac{1}{\sqrt2}\left(\hat{\bar\theta} + i\hat p_\theta\right), \qquad & \left\{\hat b, \hat b^+\right\} &= -1\,,
\end{aligned} \tag{2.10}$$

with a representation space being a tensor product of the corresponding Fock spaces, one discovers a ghost state due to the last line in eq. (2.10).

It does not seem to have been emphasized previously, that a ghost free quantum mechanics still can be constructed if we sacrifice eq. (2.9) and choose the alternative

$$\hat p_\theta^+ = -i\hat\theta\,, \qquad \hat p_{\bar\theta}^+ = -i\hat{\bar\theta}\,. \tag{2.11}$$

With such a choice, the operators

$$\begin{aligned}
\hat a &= \frac{1}{\sqrt2}\left(\hat\theta - i\hat p_{\bar\theta}\right), \qquad & \hat a^+ &= \frac{1}{\sqrt2}\left(\hat{\bar\theta} - i\hat p_\theta\right),\\
\hat b &= \frac{1}{\sqrt2}\left(\hat\theta + i\hat p_{\bar\theta}\right), \qquad & \hat b^+ &= -\frac{1}{\sqrt2}\left(\hat{\bar\theta} + i\hat p_\theta\right),
\end{aligned} \tag{2.12}$$

obey

$$\left\{\hat a, \hat a^+\right\} = 1\,, \qquad \left\{\hat b, \hat b^+\right\} = 1\,, \tag{2.13}$$

[3]In what follows we omit the indices carried by the Fermi variables.

and the corresponding Fock space, obviously, does not involve ghosts. It is worth mentioning that, there is no any physical obstruction to define a conjugation as in eq. (2.11) because eigenvalues of the Fermi operators are odd supernumbers and do not correspond to any physical observables. Notice also that the gauge fixed action (2.8) remains to be real under the modified conjugation (2.11), provided the integration by parts has been performed (one can easily check that the variation problem is not influenced by the conjugation since the fermions satisfy the first order free equations).

An explicit representation of the operators $(\hat{\theta}, \hat{\bar{\theta}}, \hat{p}_\theta, \hat{p}_{\bar{\theta}})$ in a quantum space with a scalar product respecting eq. (2.11) has been given in ref. [6] (similar issues have been discussed in ref. [20]). This is realized on a linear span of the four vectors $(|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle)$, which we collectively call $|\sigma\rangle$, with $(\hat{\theta}, \hat{\bar{\theta}}, \hat{p}_\theta, \hat{p}_{\bar{\theta}})$ operating like

$$\begin{array}{llll}
\hat{\theta}\,|0\rangle = 0\,, & \hat{\theta}\,|\uparrow\rangle = i\,|0\rangle\,, & \hat{\theta}\,|\downarrow\rangle = 0\,, & \hat{\theta}\,|\uparrow\downarrow\rangle = i\,|\downarrow\rangle\,, \\
\hat{\bar{\theta}}\,|0\rangle = 0\,, & \hat{\bar{\theta}}\,|\uparrow\rangle = 0\,, & \hat{\bar{\theta}}\,|\downarrow\rangle = i\,|0\rangle\,, & \hat{\bar{\theta}}\,|\uparrow\downarrow\rangle = -i\,|\uparrow\rangle\,, \\
\hat{p}_\theta\,|0\rangle = |\uparrow\rangle\,, & \hat{p}_\theta\,|\uparrow\rangle = 0\,, & \hat{p}_\theta\,|\downarrow\rangle = |\uparrow\downarrow\rangle\,, & \hat{p}_\theta\,|\uparrow\downarrow\rangle = 0\,, \\
\hat{p}_{\bar{\theta}}\,|0\rangle = |\downarrow\rangle\,, & \hat{p}_{\bar{\theta}}\,|\uparrow\rangle = -\,|\uparrow\downarrow\rangle\,, & \hat{p}_{\bar{\theta}}\,|\downarrow\rangle = 0\,, & \hat{p}_{\bar{\theta}}\,|\uparrow\downarrow\rangle = 0\,,
\end{array} \tag{2.14}$$

and

$$\langle\sigma|\sigma'\rangle = \delta_{\sigma\sigma'}\,. \tag{2.15}$$

The total Hilbert space is defined to be a tensor product of the linear span and the space of square integrable functions on which $\hat{x}^n$ and $\hat{p}_n$ act in the usual coordinate representation.

A physical Hilbert space in the complete quantum space is specified by the only constraint remaining

$$\hat{p}^2|\text{phys}\rangle = 0\,. \tag{2.16}$$

Restricting ourselves to momentum eigenfunction, one finds

$$\Phi_{p,\sigma} = \frac{1}{\sqrt{2(2\pi)^3}}\,|\sigma\rangle \otimes e^{-ip^0t+i\vec{p}\vec{x}}\,, \tag{2.17}$$

where for physical reasons we have chosen an upper shell of the light cone $p^0 = \sqrt{\vec{p}^2}$.

A scalar product in the physical subspace is given by

$$\langle\Phi|\Psi\rangle = i\int d^3\vec{x}\,\left(\bar{\Phi}\partial_0\Psi - \partial_0\bar{\Phi}\Psi\right), \tag{2.18}$$

or

$$\langle\Phi_{p,\sigma}|\Phi_{p',\sigma'}\rangle = p^0\delta^{(3)}\left(\vec{p}-\vec{p}'\right)\delta_{\sigma\sigma'}\,, \tag{2.19}$$

for the momentum eigenfunctions.

It is instructive then to clarify the structure of the Pauli-Lubanski vector for the case at hand. Putting the classical expression

$$W_a = \frac{1}{2}\epsilon_{abcd}\, p^b S^{cd}\,, \tag{2.20}$$

with $S^{cd} = \theta^\delta (\sigma^{cd})_\delta{}^\gamma p_{\theta\gamma} + p_{\bar{\theta}\dot{\gamma}} (\tilde{\sigma}^{cd})^{\dot{\gamma}}{}_{\dot{\delta}} \bar{\theta}^{\dot{\delta}}$ being the spin part of the Lorentz generators, onto the surface of the constraints and gauges, one obtains

$$W_a = \frac{i}{2} p_a \left(p_\theta \theta - p_{\bar{\theta}} \bar{\theta}\right). \tag{2.21}$$

Here we made use of the identities ($\epsilon_{0123} = 1$)

$$\sigma_{ab} = -\frac{i}{2}\epsilon_{abcd}\sigma^{cd}\,, \qquad \tilde{\sigma}_{ab} = \frac{i}{2}\epsilon_{abcd}\tilde{\sigma}^{cd}\,. \tag{2.22}$$

Owing to the minus sign between the two terms entering eq. (2.21), one does not face any operator ordering ambiguities in passing to quantum description. In particular,

$$\hat{W}_a \Phi_{p,\sigma} = \sigma p_a \Phi_{p,\sigma}\,, \tag{2.23}$$

where the number coefficient σ takes values

$$\sigma = \left(0, -\frac{1}{2}, \frac{1}{2}, 0\right), \tag{2.24}$$

for the states $|\sigma\rangle = (|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle)$, respectively. Observe also that $\hat{W}_a$ is hermitian with respect to both conjugation prescriptions (2.9) and (2.11).

Since the construction of unitary irreducible representations (irreps) of the Poincaré group reduces to that of the little group generated by $\hat{W}_a$ (see e.g. [21]), it is straightforward to verify that given the vector $|\sigma\rangle = (|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle)$ in eq. (2.17) the corresponding linear space (p_a takes values on the upper shell of the light cone) realizes a unitary irrep of helicity σ, with σ being specified in eq. (2.24).

Finally, it is worth mentioning, that the set of helicities obtained allows us to identify the quantum space constructed with the one particle sector of a quantized supersymmetric massless Wess-Zumino model. This correlates well with the results of the Dirac quantization[4] accomplished in ref. [6].

A path integral representation for the superpropagator of the massless Wess-Zumino model that explicitly involves a gauge fixed action of the 4d Siegel superparticle has been given in ref. [6].

[4] In ref. [6] quantum wave functions were realized on real scalar superfields. This condition can be weakened to include complex scalar superfields if one makes proper use of both of the equation entering eq. (34b) of ref. [6].

3. Siegel superparticle in an extended phase space

As was demonstrated in the previous section, the Siegel superparticle in the original formulation is infinite stage of reducibility. In this section we reformulate the model by introducing a set of auxiliary variables. The extension makes it possible to put fermionic constraints into an irreducible form valid for subsequent path integral quantization.

3.1 Action and symmetries

The superparticle action to be analyzed is

$$S = \int d\tau \left\{ \frac{1}{2e}\left(\dot{x}^m + i\theta\sigma^m\dot{\bar{\theta}} - i\dot{\theta}\sigma^m\bar{\theta} + i\psi\sigma^m\bar{\rho} - i\rho\sigma^m\bar{\psi} + \omega\Lambda^m\right)^2 - \right.$$
$$\left. - \rho^\alpha\dot{\theta}_\alpha - \bar{\rho}_{\dot{\alpha}}\dot{\bar{\theta}}^{\dot{\alpha}} - \omega - \phi\Lambda^2 - \Lambda_m i\varphi\sigma^m\bar{\chi} + \Lambda_m i\chi\sigma^m\bar{\varphi} \right\}. \tag{3.1}$$

The theory is invariant under the standard rigid supersymmetry transformations. The local reparametrizations and κ-symmetry of Siegel's model

$$\begin{aligned}
&\delta_\alpha\theta = \alpha\dot{\theta}\,, && \delta_\alpha\bar{\theta} = \alpha\dot{\bar{\theta}}\,, && \delta_\alpha x^n = \alpha\dot{x}^n\,,\\
&\delta_\alpha\rho = \alpha\dot{\rho}\,, && \delta_\alpha\bar{\rho} = \alpha\dot{\bar{\rho}}\,, && \delta_\alpha e = (\alpha e)^{\cdot}\,,\\
&\delta_\alpha\psi = (\alpha\psi)^{\cdot}\,, && \delta_\alpha\bar{\psi} = (\alpha\bar{\psi})^{\cdot}\,, && \delta_\alpha\omega = (\alpha\omega)^{\cdot}\,,\\
&\delta_\alpha\Lambda^n = \alpha\dot{\Lambda}^n\,, && \delta_\alpha\chi = \alpha\dot{\chi}\,, && \delta_\alpha\bar{\chi} = \alpha\dot{\bar{\chi}}\,,\\
&\delta_\alpha\varphi = (\alpha\varphi)^{\cdot}\,, && \delta_\alpha\bar{\varphi} = (\alpha\bar{\varphi})^{\cdot}\,, && \delta_\alpha\phi = (\alpha\phi)^{\cdot}\,,\\
&\delta_\kappa\theta = -ie^{-1}\Pi_n\sigma^n\bar{\kappa}\,, && \delta_\kappa\bar{\theta} = ie^{-1}\Pi_n\kappa\sigma^n\,, &&\\
&\delta_\kappa x^n = i\delta_\kappa\theta\sigma^n\bar{\theta} - i\theta\sigma^n\delta_\kappa\bar{\theta} - i\kappa\sigma^n\bar{\rho} + i\rho\sigma^n\bar{\kappa}\,, &&&&\\
&\delta_\kappa e = 4\dot{\theta}\kappa + 4\bar{\kappa}\dot{\bar{\theta}}\,, && \delta_\kappa\psi = \dot{\kappa}\,, && \delta_\kappa\bar{\psi} = \dot{\bar{\kappa}}\,,
\end{aligned} \tag{3.2}$$

where $\Pi^m = \dot{x}^m + i\theta\sigma^m\dot{\bar{\theta}} - i\dot{\theta}\sigma^m\bar{\theta} + i\psi\sigma^m\bar{\rho} - i\rho\sigma^m\bar{\psi} + \omega\Lambda^m$, are extended by two new symmetries depending on fermionic parameters β, γ, the latter acting in the sector of the new variables

$$\delta_\beta\chi = \bar{\beta}\tilde{\sigma}^n\Lambda_n\,, \qquad \delta_\beta\bar{\chi} = \Lambda_n\tilde{\sigma}^n\beta\,, \qquad \delta_\beta\phi = i\left(\varphi\beta - \bar{\varphi}\bar{\beta}\right), \tag{3.3}$$

$$\delta_\gamma\varphi = \bar{\gamma}\tilde{\sigma}^n\Lambda_n\,, \qquad \delta_\gamma\bar{\varphi} = \Lambda_n\tilde{\sigma}^n\gamma\,, \qquad \delta_\gamma\phi = -i\left(\chi\gamma - \bar{\chi}\bar{\gamma}\right). \tag{3.4}$$

From the transformation rules above, one concludes that the variables $(x^m, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$ parametrize a conventional $R^{4|4}$ superspace, $(e, \psi^\alpha, \bar{\psi}_{\dot{\alpha}})$ prove to be gauge fields for local reparametrizations and κ-symmetry, whereas the pair $(\rho^\alpha, \bar{\rho}_{\dot{\alpha}})$ provides the terms corresponding to a (mixed) covariant propagator for fermions. This holds as in the Siegel model. As shown below, there is no dynamics in the sector of the new variables $(\omega, \Lambda^m, \phi, \varphi^\alpha, \bar{\varphi}_{\dot{\alpha}}, \chi^\alpha, \bar{\chi}_{\dot{\alpha}})$, these prove to be purely auxiliary.

3.2 Fermionic constraints made irreducible

Proceeding to the hamiltonian analysis one finds fourteen primary constraints[5]

$$\begin{aligned} &p_e = 0\,, \quad p_\psi = 0\,, \quad p_{\bar\psi} = 0\,, \quad p_\rho = 0\,, \quad p_{\bar\rho} = 0\,, \quad p_\omega = 0\,, \\ &p_\Lambda = 0\,, \quad p_\phi = 0\,, \quad p_\varphi = 0\,, \quad p_{\bar\varphi} = 0\,, \quad p_\chi = 0\,, \quad p_{\bar\chi} = 0\,, \end{aligned} \tag{3.5}$$

$$p_{\theta\alpha} - p_n i(\sigma^n\bar\theta)_\alpha - \rho_\alpha = 0\,, \qquad p_{\bar\theta}{}^{\dot\alpha} + p_n i(\theta\sigma^n)^{\dot\alpha} - \bar\rho^{\dot\alpha} = 0\,, \tag{3.6}$$

where p_q stands for a momentum canonically conjugate to a variable q. The total hamiltonian has the form

$$\begin{aligned} H = {} & p_e\lambda_e + p_{\psi\alpha}\lambda_\psi{}^\alpha + p_{\bar\psi}{}^{\dot\alpha}\lambda_{\bar\psi\dot\alpha} + p_{\rho\alpha}\lambda_\rho{}^\alpha + p_{\bar\rho}{}^{\dot\alpha}\lambda_{\bar\rho\dot\alpha} + p_\omega\lambda_\omega + p_{\Lambda n}\lambda_\Lambda{}^n + p_\phi\lambda_\phi + \\ & + p_{\varphi\alpha}\lambda_\varphi{}^\alpha + p_{\bar\varphi}{}^{\dot\alpha}\lambda_{\bar\varphi\dot\alpha} + p_{\chi\alpha}\lambda_\chi{}^\alpha + p_{\bar\chi}{}^{\dot\alpha}\lambda_{\bar\chi\dot\alpha} + (p_{\bar\theta} + p_n i\theta\sigma^n - \bar\rho)^{\dot\alpha}\lambda_{\bar\theta\dot\alpha} + \\ & + (p_\theta - p_n i\sigma^n\bar\theta - \rho)_\alpha\lambda_\theta{}^\alpha + \frac{1}{2}ep^2 - i\psi\sigma^n\bar\rho p_n + \\ & + i\rho\sigma^n\bar\psi p_n + \phi\Lambda^2 + \omega(1-\Lambda p) + i\varphi\sigma^n\bar\chi\Lambda_n - i\chi\sigma^n\bar\varphi\Lambda_n\,, \end{aligned} \tag{3.7}$$

where $\lambda_{\ldots}$ are the Lagrange multipliers associated to the primary constraints. The conservation in time of the primary constraints yields the secondary ones

$$\begin{aligned} &p^2 = 0\,, \quad p_n(\sigma^n\bar\rho)_\alpha = 0\,, \quad p_n(\rho\sigma^n)_{\dot\alpha} = 0\,, \\ &\Lambda_n(\sigma^n\bar\chi)_\alpha = 0\,, \quad \Lambda_n(\chi\sigma^n)_{\dot\alpha} = 0\,, \quad \Lambda_n(\sigma^n\bar\varphi)_\alpha = 0\,, \quad \Lambda_n(\varphi\sigma^n)_{\dot\alpha} = 0\,, \\ &\Lambda^2 = 0\,, \quad 1-\Lambda p = 0\,, \quad -2\phi\Lambda^n + \omega p^n - i\varphi\sigma^n\bar\chi + i\chi\sigma^n\bar\varphi = 0\,, \end{aligned} \tag{3.8}$$

and fixes some of the Lagrange multipliers,

$$\begin{aligned} &\lambda_\theta = -p_n i\sigma^n\bar\psi\,, \qquad \lambda_{\bar\theta} = i\psi\sigma^n p_n\,, \\ &\lambda_\rho = -2p_n i\sigma^n\lambda_{\bar\theta} \approx 0\,, \qquad \lambda_{\bar\rho} = 2i\lambda_\theta\sigma^n p_n \approx 0\,. \end{aligned} \tag{3.9}$$

Beautifully enough, the last equation in eq. (3.8) can be simplified to (a proof is given in appendix A)

$$\omega = 0\,, \qquad -2\phi - i\varphi\sigma^n\bar\chi p_n + i\chi\sigma^n\bar\varphi p_n = 0\,. \tag{3.10}$$

With this remark, consistency conditions for the secondary constraints amount to

$$\begin{aligned} &p\lambda_\Lambda = 0\,, \quad \Lambda\lambda_\Lambda = 0\,, \quad \lambda_\omega = 0\,, \\ &2\lambda_\phi = -i\lambda_\varphi\sigma^n\bar\chi p_n + i\lambda_\chi\sigma^n\bar\varphi p_n - i\varphi\sigma^n\lambda_{\bar\chi}p_n + i\chi\sigma^n\lambda_{\bar\varphi}p_n\,, \\ &\Lambda_n(\sigma^n\lambda_{\bar\chi})_\alpha + \lambda_{\Lambda n}(\sigma^n\bar\chi)_\alpha = 0\,, \quad \Lambda_n(\lambda_\chi\sigma^n)_{\dot\alpha} + \lambda_{\Lambda n}(\chi\sigma^n)_{\dot\alpha} = 0\,, \\ &\Lambda_n(\sigma^n\lambda_{\bar\varphi})_\alpha + \lambda_{\Lambda n}(\sigma^n\bar\varphi)_\alpha = 0\,, \quad \Lambda_n(\lambda_\varphi\sigma^n)_{\dot\alpha} + \lambda_{\Lambda n}(\varphi\sigma^n)_{\dot\alpha} = 0\,. \end{aligned} \tag{3.11}$$

[5] We define momenta conjugate to Fermi variables to be right derivatives of a lagrangian with respect to velocities. This corresponds to the following choice of the Poisson brackets $\{\theta^\alpha, p_{\theta\beta}\} = \delta^\alpha{}_\beta, \{\bar\theta_{\dot\alpha}, p_{\bar\theta}{}^{\dot\beta}\} = \delta_{\dot\alpha}{}^{\dot\beta}$ and the position of momenta and velocities in the hamiltonian as specified below in eq. (3.7). Our conventions for the conjugation slightly differ from those used in the review section $(\theta^\alpha)^* = \bar\theta^{\dot\alpha}$, $(p_{\theta\alpha})^* = p_{\bar\theta\dot\alpha}$.

Making use of light-cone arguments like those given in the Appendix one can show that each of the fermionic equations entering eq. (3.11) determines precisely half of the corresponding fermionic Lagrange multipliers.

Thus no tertiary constraints arise at this stage, the complete constraint system being

$$p_e = 0\,, \qquad p_\psi = 0\,, \qquad p_{\bar\psi} = 0\,, \tag{3.12}$$

$$p_\rho = 0\,, \qquad p_\theta - p_n i(\sigma^n\bar\theta) - \rho = 0\,, \tag{3.13}$$

$$p_{\bar\rho} = 0\,, \qquad p_{\bar\theta} + p_n i(\theta\sigma^n) - \bar\rho = 0\,, \tag{3.14}$$

$$p_\omega = 0\,, \qquad \omega = 0\,, \tag{3.15}$$

$$p_\phi = 0\,, \qquad -2\phi - i\varphi\sigma^n\bar\chi p_n + i\chi\sigma^n\bar\varphi p_n = 0\,, \tag{3.16}$$

$$p_\varphi = 0\,, \qquad \varphi\sigma^n\Lambda_n = 0\,, \tag{3.17}$$

$$p_{\bar\varphi} = 0\,, \qquad \sigma^n\bar\varphi\Lambda_n = 0\,, \tag{3.18}$$

$$p_\chi = 0\,, \qquad \chi\sigma^n\Lambda_n = 0\,, \tag{3.19}$$

$$p_{\bar\chi} = 0\,, \qquad \sigma^n\bar\chi\Lambda_n = 0\,, \tag{3.20}$$

$$p^2 = 0\,, \qquad p_\theta\sigma^n p_n = 0\,, \qquad \sigma^n p_{\bar\theta} p_n = 0\,, \tag{3.21}$$

$$p_\Lambda = 0\,, \qquad \Lambda^2 = 0\,, \qquad 1 - \Lambda p = 0\,. \tag{3.22}$$

The constraints (3.12) are first-class. Imposing the gauge

$$e = 1\,, \qquad \psi = 0\,, \qquad \bar\psi = 0\,, \tag{3.23}$$

which yields

$$\lambda_e = 0\,, \qquad \lambda_\psi = 0\,, \qquad \lambda_{\bar\psi} = 0\,, \tag{3.24}$$

one can disregard the canonical pairs (e, p_e), (ψ, p_ψ), $(\bar\psi, p_{\bar\psi})$. In the same manner, the variables (ρ, p_ρ), $(\bar\rho, p_{\bar\rho})$, (ω, p_ω), (ϕ, p_ϕ) can be omitted after introducing the Dirac bracket associated with the second class constraints (3.13)–(3.16). The Dirac brackets for the remaining variables prove to coincide with the Poisson ones.

One has to be more inventive when imposing a gauge in the sector (3.17) and (3.18). Passing to the light-cone coordinates (see appendix A) one concludes that, due to $\Lambda^2 = 0$, there is only one linearly independent component entering the last of the spinor constraints (3.17) and (3.18), this proves to be second class, whereas the corresponding momenta include one first and one second class constraints. Beautifully enough, on account of the last of the equations (3.22) these can be put into covariant (redundant) form

$$p_\varphi = 0 \Longleftrightarrow \begin{cases} p_\varphi\sigma^n\Lambda_n = 0 & \text{first class} \\ p_\varphi\sigma^n p_n = 0 & \text{second class}\,. \end{cases} \tag{3.25}$$

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Fixing a gauge is now obvious (again in a covariant and redundant form)

$$\sigma^n \varphi p_n = 0\,, \tag{3.26}$$

which yields

$$\varphi = 0\,, \tag{3.27}$$

when combined with eq. (3.17). The conservation in time of the gauge (3.26) yields

$$\lambda_\varphi \sigma^n p_n = 0\,. \tag{3.28}$$

Together with eq. (3.11) this completely specifies λ_φ. Note also that consistency ($(\varphi)^* = \bar{\varphi}$) requires us to impose the complex conjugate equation

$$p_n \sigma^n \bar{\varphi} = 0 \longrightarrow \bar{\varphi} = 0\,. \tag{3.29}$$

One finally concludes that there is no dynamics in the sector (φ, p_φ), $(\bar{\varphi}, p_{\bar{\varphi}})$.

The same arguments apply to the variables (χ, p_χ), $(\bar{\chi}, p_{\bar{\chi}})$. For our purposes, however, it is convenient not to impose a gauge in this sector but rather use these purely auxiliary variables to supplement Siegel's constraints (3.21) up to irreducible ones. Actually, it is straightforward to check that the system (see also ref. [17])

$$\begin{aligned}
\bar{\Phi}_{\dot{\alpha}} &\equiv (p_\theta \sigma^n p_n + p_\chi \sigma^n \Lambda_n)_{\dot{\alpha}} = 0\,, \\
\Phi_\alpha &\equiv (p_n \sigma^n p_{\bar{\theta}} + \Lambda_n \sigma^n p_{\bar{\chi}})_\alpha = 0 \qquad \text{first class}\,,
\end{aligned} \tag{3.30}$$

$$\begin{aligned}
\bar{\Psi}_{\dot{\alpha}} &\equiv (\chi \sigma^n \Lambda_n + p_\chi \sigma^n p_n)_{\dot{\alpha}} = 0\,, \\
\Psi_\alpha &\equiv (\Lambda_n \sigma^n \bar{\chi} + p_n \sigma^n p_{\bar{\chi}})_\alpha = 0 \qquad \text{second class}\,,
\end{aligned} \tag{3.31}$$

$$p^2 = 0 \qquad \text{first class}\,, \tag{3.32}$$

is completely equivalent to the initial equations (3.19)–(3.21). Here the identities

$$p_\chi{}^\alpha = -\frac{1}{2\Lambda p}\bar{\Phi}_{\dot{\alpha}}(\tilde{\sigma}^m p_m)^{\dot{\alpha}\alpha} - \frac{1}{2\Lambda p}\bar{\Psi}_{\dot{\alpha}}(\tilde{\sigma}^m \Lambda_m)^{\dot{\alpha}\alpha} - \frac{1}{2\Lambda p} p^2 p_\theta{}^\alpha - \frac{1}{2\Lambda p}\Lambda^2 \chi^\alpha\,, \tag{3.33}$$

$$p_{\bar{\chi}}{}^{\dot{\alpha}} = -\frac{1}{2\Lambda p}(\tilde{\sigma}^m p_m)^{\dot{\alpha}\alpha}\Phi_\alpha - \frac{1}{2\Lambda p}(\tilde{\sigma}^m \Lambda_m)^{\dot{\alpha}\alpha}\Psi_\alpha - \frac{1}{2\Lambda p} p^2 p_{\bar{\theta}}{}^{\dot{\alpha}} - \frac{1}{2\Lambda p}\Lambda^2 \bar{\chi}^{\dot{\alpha}}\,, \tag{3.34}$$

prove to be useful. The equivalence just stated implies also that the constraint set above is *irreducible*, otherwise we would have less than $8+1$ equations and eqs. (3.30)–(3.32) would not be equivalent to (3.19)–(3.21) (8+1 linearly independent components).

It remains to discuss the bosonic constraints (3.22). Constructing a (weak) projector to the directions orthogonal to the vectors p^n, Λ^n

$$\pi_m{}^n = \delta_m{}^n - p_m \Lambda^n - \Lambda_m p^n\,, \tag{3.35}$$

one can easily extract first class constraints contained in p_Λ, the complete constraint set being

$$\tilde{p}_{\Lambda m} \equiv (\pi p_\Lambda)_m = p_{\Lambda m} - (p_\Lambda \Lambda) p_m - (p_\Lambda p)\Lambda_m = 0 \qquad \text{first class}\,, \tag{3.36}$$

$$p_\Lambda p = 0\,, \qquad \Lambda^2 = 0\,, \qquad p_\Lambda \Lambda = 0\,, \qquad 1 - \Lambda p = 0 \qquad \text{second class}\,. \tag{3.37}$$

In view of the identities[6]

$$\tilde{p}_\Lambda \Lambda \approx 0\,, \qquad \tilde{p}_\Lambda p \approx 0\,, \tag{3.38}$$

one concludes that there are only two linearly independent components entering eq. (3.36), the total number of constraints being sufficient to suppress dynamics in the sector. In order to explicitly decouple the first class constraints above from the fermionic second class ones (3.31), it suffices to redefine them like

$$\tilde{p}^n_\Lambda = 0 \longrightarrow \tilde{p}^n_\Lambda - \frac{1}{2}\chi\sigma^n\tilde{\sigma}^m p_\chi p_m - \frac{1}{2} p_{\bar\chi}\tilde{\sigma}^m\sigma^n\bar\chi p_m = 0\,. \tag{3.39}$$

As the Dirac bracket associated with the second class constraints is introduced, this seems to be inessential here.

It is worth mentioning, that the dynamical equivalence of the model (3.1) and the Siegel superparticle [1] can be easily established if one imposes the non-covariant gauge

$$\Lambda^i = 0\,, \qquad i = 1, 2\,. \tag{3.40}$$

To summarize, in the extended phase space the infinite reducibility of the constraints (3.21) characterizing the Siegel model can be compensated by that coming from the sector of additional variables to put the fermionic constraints into an irreducible form. Residual reducibility proves to fall in the bosonic constraints (3.36) and (3.37). Being the first stage of reducibility, these admit consistent path integral quantization.

Quantization of the constraint system (3.30)–(3.32), (3.36) and (3.37) will be our main concern in the next sections.

3.3 The Dirac bracket

In the presence of second class constraints both the nilpotency equation to determine the BRST charge and that to fix the unitarizing hamiltonian should be solved under the Dirac bracket associated with the full set of second class constraints [18]. To construct the latter, it suffices to convert the matrix of Poisson brackets of second class constraints.[7] Denoting the constraints collectively by $\Theta_i = (p_\Lambda p, \Lambda^2, p_\Lambda \Lambda, 1 -$

[6]Here and in what follows the symbol $\approx$ means an equality up to a linear combination of *second class* constraints.

[7]The construction proves to be more involved when second class constraints in a question are (infinitely) reducible. A recipe has been given in ref. [22]

$\Lambda p, \bar{\Psi}_{\dot\alpha}, \Psi_\alpha)$ and $\Gamma_{ij} \equiv \{\Theta_i, \Theta_j\}$, one finds this to be

$$\Gamma_{ij} = \tag{3.41}$$
$$= \begin{pmatrix} 0 & -2\Lambda p & -pp_\Lambda & p^2 & -(\chi\sigma^n p_n)_{\dot\beta} & -(p_n\sigma^n\bar\chi)_\beta \\ 2\Lambda p & 0 & 2\Lambda^2 & 0 & 0 & 0 \\ pp_\Lambda & -2\Lambda^2 & 0 & \Lambda p & -(\chi\sigma^n\Lambda_n)_{\dot\beta} & -(\Lambda_n\sigma^n\bar\chi)_\beta \\ -p^2 & 0 & -\Lambda p & 0 & 0 & 0 \\ (\chi\sigma^n p_n)_{\dot\alpha} & 0 & (\chi\sigma^n\Lambda_n)_{\dot\alpha} & 0 & -4(\tilde\sigma^{nm})_{\dot\alpha\dot\beta}\Lambda_n p_m & 0 \\ (p_n\sigma^n\bar\chi)_\alpha & 0 & (\Lambda_n\sigma^n\bar\chi)_\alpha & 0 & 0 & -4(\sigma^{nm})_{\alpha\beta}\Lambda_n p_m \end{pmatrix}.$$

The corresponding superdeterminant amounts to a simple number coefficient

$$\mathrm{sdet}\,\Gamma_{ij} = \frac{1}{4}\,, \tag{3.42}$$

this to be used when constructing the path integral measure in section 5.

Given a supermatrix $F = F_B + F_S$, where F_B and F_S are the body and the soul, respectively [23], the inverse supermatrix is constructed according to the rule [23]

$$F^{-1} = F_B{}^{-1} + \sum_{k=1}^{\infty}(-1)^k (F_B{}^{-1}F_S)^k F_B{}^{-1}\,. \tag{3.43}$$

In our case only the first two terms entering the power series above prove to be non vanishing, the corresponding inverse supermatrix being

$$\Gamma^{ij} = \frac{1}{\Delta}\times \tag{3.44}$$
$$\times\begin{pmatrix} 0 & \Lambda p & 0 & 2\Lambda^2 & 0 & 0 \\ -\Lambda p & 0 & p^2 & pp_\Lambda{}' & -\frac{1}{2}(\chi\sigma^n p_n)^{\dot\beta} & \frac{1}{2}(p_n\sigma^n\bar\chi)^\beta \\ 0 & -p^2 & 0 & -2\Lambda p & 0 & 0 \\ -2\Lambda^2 & -pp_\Lambda{}' & 2\Lambda p & 0 & -(\chi\sigma^n\Lambda_n)^{\dot\beta} & (\Lambda_n\sigma^n\bar\chi)^\beta \\ 0 & -\frac{1}{2}(\chi\sigma^n p_n)^{\dot\alpha} & 0 & -(\chi\sigma^n\Lambda_n)^{\dot\alpha} & 2(\tilde\sigma^{nm})^{\dot\alpha\dot\beta}\Lambda_n p_m & 0 \\ 0 & \frac{1}{2}(p_n\sigma^n\bar\chi)^\alpha & 0 & (\Lambda_n\sigma^n\bar\chi)^\alpha & 0 & 2(\sigma^{nm})^{\alpha\beta}\Lambda_n p_m \end{pmatrix},$$

where $\Delta \equiv 2((\Lambda p)^2 - \Lambda^2 p^2)$ and $pp_\Lambda{}' \equiv pp_\Lambda + \frac{1}{2}(\chi^2 + \bar\chi^2)$.

With the Γ^{ij} at hand, the Dirac bracket is straightforward to build

$$\{A, B\}_D = \{A, B\} - \{A, \Theta_i\}\Gamma^{ij}\{\Theta_j, B\}\,. \tag{3.45}$$

Being rather involved in the general form, the bracket considerably simplifies when evaluated in specific coordinate sectors[8] (only the brackets to be used below are explicitly given here)

$$\{\chi^\alpha, p_{\chi\beta}\} = \frac{1}{2}\delta^\alpha{}_\beta - \frac{2}{\Delta}\Lambda p(\sigma_{nm})_\beta{}^\alpha\Lambda^n p^m\,, \qquad \{\chi^\alpha, \chi^\beta\} = \frac{2}{\Delta}p^2(\sigma_{nm})^{\alpha\beta}\Lambda^n p^m\,,$$

[8]In what follows we omit the label D attached to the Dirac brackets.

$$
\begin{aligned}
\{p_{\chi\alpha}, p_{\chi\beta}\} &= \frac{2}{\Delta}\Lambda^2(\sigma_{nm})_{\alpha\beta}\Lambda^n p^m\,;\\
\{\Lambda^n, p_{\Lambda m}\} &= \delta^n{}_m - \frac{2}{\Delta}\Lambda p(p^n\Lambda_m + \Lambda^n p_m) + \frac{2}{\Delta}p^2\Lambda^n\Lambda_m + \frac{2}{\Delta}\Lambda^2 p^n p_m\,,\\
\{\Lambda^n, \Lambda^m\} &= 0\,,\\
\{p_{\Lambda n}, p_{\Lambda m}\} &= \frac{2}{\Delta}p^2(\Lambda_n p_{\Lambda m} - \Lambda_m p_{\Lambda n}) + \frac{2}{\Delta}pp_\Lambda(p_n\Lambda_m - p_m\Lambda_n) +\\
&\quad + \frac{2}{\Delta}p\Lambda(p_{\Lambda n}p_m - p_{\Lambda m}p_n) - \frac{i}{\Delta}(\chi^2 - \bar{\chi}^2)\epsilon_{nmkl}\Lambda^k p^l\,; \qquad (3.46)\\
\{\theta^\alpha, p_{\theta\beta}\} &= \delta^\alpha{}_\beta\,, \qquad \{\theta^\alpha, \theta^\beta\} = 0\,, \qquad \{p_{\theta\alpha}, p_{\theta\beta}\} = 0\,, \qquad \{p_n, p_m\} = 0\,.
\end{aligned}
$$

Analogously, for the cross sectors one finds (in what follows we will not need the explicit form of the brackets involving the x^n-variable, these are omitted here)

$$
\begin{aligned}
\{p_{\Lambda n}, \chi^\alpha\} &= \frac{1}{\Delta}p^2\Lambda_n\chi^\alpha + \frac{1}{\Delta}p^2\big(\chi\sigma_n\tilde{\sigma}^k\Lambda_k\big)^\alpha + \frac{1}{\Delta}p_n\big(\chi\sigma^k\Lambda_k\tilde{\sigma}^m p_m\big)^\alpha -\\
&\quad - \frac{1}{\Delta}\Lambda p\big(\chi\sigma_n\tilde{\sigma}^k p_k\big)^\alpha,\\
\{p_{\Lambda n}, p_{\chi\alpha}\} &= \frac{1}{\Delta}\Lambda^2 p_n\chi_\alpha + \frac{1}{\Delta}\Lambda^2\big(\chi\sigma_n\tilde{\sigma}^k p_k\big)_\alpha + \frac{1}{\Delta}\Lambda_n\big(\chi\sigma^k p_k\tilde{\sigma}^m\Lambda_m\big)_\alpha -\\
&\quad - \frac{1}{\Delta}\Lambda p\big(\chi\sigma_n\tilde{\sigma}^k\Lambda_k\big)_\alpha\,. \qquad (3.47)
\end{aligned}
$$

Similar relations hold for complex conjugates. In particular, to derive brackets involving $(\bar{\chi}, p_{\bar{\chi}})$ it suffices to raise α, low β and then exchange them with dotted indices.

In obtaining eqs. (3.46) and (3.47) the following identities:

$$
\begin{aligned}
\mathrm{Tr}(\sigma_{ab}\sigma_{cd}) &= -\frac{1}{2}(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) + \frac{i}{2}\epsilon_{abcd}\,,\\
\mathrm{Tr}(\tilde{\sigma}_{ab}\tilde{\sigma}_{cd}) &= -\frac{1}{2}(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) - \frac{i}{2}\epsilon_{abcd}\,, \qquad (3.48)
\end{aligned}
$$

with $\epsilon_{0123} = 1$ and $\eta_{nm} = \mathrm{diag}(-,+,+,+)$, prove to be useful.

Finally, it is worth noting that, as long as the path integral quantization is concerned, the presence of the δ-function of second class constraints in the path integral measure [18] allows one to solve the equations on the BRST-charge and the unitarizing hamiltonian modulo second class constraints [18]. In particular, this will provide further simplifications in eqs. (3.46)–(3.47).

3.4 The algebra of first class constraints

Having evaluated the Dirac bracket, we are now in a position to specify the algebra of the first class constraints (3.30), (3.32) and (3.36), the corresponding structure functions to be used when constructing the BRST charge.

Taking into account eq. (3.33), the identity

$$\chi^2 = -\frac{1}{\Lambda p}p^2(p_\chi \chi) - \frac{1}{\Lambda p}\bar{\Psi}_{\dot\alpha}(\tilde\sigma^m p_m \chi)^{\dot\alpha}\,, \tag{3.49}$$

and the fact that according to the general recipe [18] it suffices to know the algebra modulo second class constraints, one finds the only non-trivial brackets to be

$$\begin{aligned}
&\{\tilde p_{\Lambda n}, \tilde p_{\Lambda m}\} \approx U_{nm}{}^k \tilde p_{\Lambda k} + U_{nm} p^2\,, \qquad \{\tilde p_{\Lambda n}, \Phi_\alpha\} \approx U_{n\alpha}{}^\beta \Phi_\beta + U_{n\alpha} p^2\,,\\
&\{\tilde p_{\Lambda n}, \bar\Phi_{\dot\alpha}\} \approx U_{n\dot\alpha}{}^{\dot\beta}\bar\Phi_{\dot\beta} + U_{n\dot\alpha}p^2\,.
\end{aligned} \tag{3.50}$$

The explicit form of the structure functions involved is ($U_{n\dot\alpha}{}^{\dot\beta}$, $U_{n\dot\alpha}$ are obtained by complex conjugation)

$$\begin{aligned}
U_{nm}{}^k &= \frac{2}{\Delta}\Big(\big(\Lambda_n p^2 - p_n\big)\,\delta_m{}^k - \big(\Lambda_m p^2 - p_m\big)\,\delta_n{}^k\Big)\,,\\
U_{nm} &= \frac{i}{\Delta}(p_\chi \chi - p_{\bar\chi}\bar\chi)\epsilon_{nmkl}\Lambda^k p^l\,,\\
U_{n\alpha}{}^\beta &= \frac{1}{2}\big(\sigma_n \tilde\sigma^k p_k\big)_\alpha{}^\beta + \frac{1}{\Delta}\Lambda_n p^2 \delta_\alpha{}^\beta + \frac{1}{\Delta}\,\big(\Lambda_n p^2 - p_n\big)\,\big(\Lambda^k \sigma_k \tilde\sigma^l p_l\big)_\alpha{}^\beta\,,\\
U_{n\alpha} &= \frac{1}{2}(\sigma_n p_{\bar\theta})_\alpha - \frac{1}{\Delta}\Lambda_n\big(p^k \sigma_k p_{\bar\theta}\big)_\alpha + \frac{1}{\Delta}\,\big(\Lambda_n p^2 - p_n\big)\,\big(\Lambda^k \sigma_k p_{\bar\theta}\big)_\alpha\,.
\end{aligned} \tag{3.51}$$

Worth noting also are the important algebraic properties of the functions obtained (the same holds for complex conjugates)

$$\begin{aligned}
U_{nm}\Lambda^m &= 0\,, \qquad U_{n\alpha}{}^\beta \Lambda^n \approx 0\,, \qquad U_{n\alpha}\Lambda^n \approx 0\,,\\
U_{nm}p^m &= 0\,, \qquad U_{n\alpha}{}^\beta p^n \approx 0\,, \qquad U_{n\alpha}p^n \approx 0\,.
\end{aligned} \tag{3.52}$$

These will be of frequent use when establishing the nilpotency of the BRST charge in the next section.

4. The BRST charge and the unitarizing hamiltonian in the minimal ghost sector

Proceeding to the BRST quantization, one associates a couple of canonically conjugate ghost variables to each of the first class constraints (3.30), (3.32) and (3.36) $(C^{\dot\alpha}, \bar{\mathcal P}_{\dot\alpha})$, $(C^\alpha, \bar{\mathcal P}_\alpha)$, $(C, \bar{\mathcal P})$, $(C^n, \bar{\mathcal P}_n)$. The statistics and the ghost number are specified by the conventional prescriptions

$$\epsilon(C^A) = \epsilon(\bar{\mathcal P}^A) = \epsilon_A + 1\,, \qquad gh(C^A) = -gh(\bar{\mathcal P}^A) = 1\,. \tag{4.1}$$

To compensate the overcounting in the sector $(C^n, \bar{\mathcal P}_n)$ (only two components entering the bosonic constraint (3.36) are linearly independent) one further introduces [18] the secondary ghosts $(C^1, \bar{\mathcal P}^1)$, $(C^2, \bar{\mathcal P}^2)$ which obey

$$\epsilon(C^{1,2}) = \epsilon(\bar{\mathcal P}^{1,2}) = 0\,, \qquad gh(C^{1,2}) = -gh(\bar{\mathcal P}^{1,2}) = 2\,. \tag{4.2}$$

Together with the previously introduced variables these exhaust the minimal ghost sector for the model under consideration.

The BRST charge is defined to be a solution of the nilpotency equation

$$\{\Omega_{\rm min}, \Omega_{\rm min}\} \approx 0\,, \tag{4.3}$$

satisfying the boundary condition

$$\Omega_{\rm min} = \Phi_\alpha C^\alpha + \bar{\Phi}_{\dot\alpha} C^{\dot\alpha} + \tilde{p}_{\Lambda n} C^n + p^2 C + \bar{\mathcal{P}}_n \Lambda^n C^1 + \bar{\mathcal{P}}_n p^n C^2 + \cdots. \tag{4.4}$$

The first four terms entering eq. (4.4) are typical for the BRST quantization of irreducible gauge theories. Through eq. (4.3) they automatically generate the gauge algebra (3.50). The two remaining terms are designed to generate the identities (3.38) and are specific to the treatment of reducible theories.

Calculating the contribution of the boundary terms into eq. (4.3)

$$\begin{aligned}\{\Omega_{\rm min}, \Omega_{\rm min}\} \approx{}& 2\bar{\mathcal{P}}_m \left\{\Lambda^m, \tilde{p}_{\Lambda n}\right\} C^1 C^n - 2\left(U_{n\alpha}{}^\beta \Phi_\beta + U_{n\alpha} p^2\right) C^\alpha C^n - \\ & - 2\left(U_{n\dot\alpha}{}^{\dot\beta} \bar{\Phi}_{\dot\beta} + U_{n\dot\alpha} p^2\right) C^{\dot\alpha} C^n - \left(U_{nm}{}^k \tilde{p}_{\Lambda k} + U_{nm} p^2\right) C^m C^n + \cdots,\end{aligned} \tag{4.5}$$

one can partially clarify the structure of the terms which were missing in eq. (4.4). In particular, extending the ansatz (4.4) by means of three new contributions

$$\frac{1}{2}\bar{\mathcal{P}}_k \tilde{U}^k_{nm} C^m C^n + \bar{\mathcal{P}}_\alpha U_{n\beta}{}^\alpha C^\beta C^n + \bar{\mathcal{P}}_{\dot\alpha} U_{n\dot\beta}{}^{\dot\alpha} C^{\dot\beta} C^n\,, \tag{4.6}$$

where

$$\begin{aligned}\tilde{U}^k_{nm} &= U_{nm}{}^k - \frac{2}{\Delta} p^k \left(\Lambda_n p_m - \Lambda_m p_n\right), \\ \tilde{U}^k_{nm} \Lambda^m &\approx \frac{2}{\Delta}\left\{\Lambda^k, p_{\Lambda n}\right\}, \qquad \tilde{U}^k_{nm} p^m \approx 0\,,\end{aligned} \tag{4.7}$$

one can get rid of the first term (which is a manifestation of the reducibility of the constraints) and those involving $\tilde{p}_\Lambda, \Phi, \bar{\Phi}$

$$\begin{aligned}\{\Omega_{\rm min}, \Omega_{\rm min}\} \approx{}& -U_{nm} p^2 C^m C^n - 2U_{n\alpha} p^2 C^\alpha C^n - 2U_{n\dot\alpha} p^2 C^{\dot\alpha} C^n - \\ & - 2\bar{\mathcal{P}}_\alpha U_{n\gamma}{}^\alpha U_{m\beta}{}^\gamma C^m C^n C^\beta - 2\bar{\mathcal{P}}_{\dot\alpha} U_{n\dot\gamma}{}^{\dot\alpha} U_{m\dot\beta}{}^{\dot\gamma} C^m C^n C^{\dot\beta} + \cdots.\end{aligned} \tag{4.8}$$

In order to verify eq. (4.8) one has to use the algebraic properties of the structure functions (3.52) and the Jacobi identities resulting from the constraint algebra

$$\begin{aligned}&\tilde{U}^k_{[m\hat{a}} \tilde{U}^a_{bc]} \approx 0\,, \qquad U_{[m\hat{a}}{}^k U_{bc]}{}^a \approx 0\,, \qquad \{\tilde{p}_{\Lambda[n}, \tilde{U}^k_{ab]}\} \approx 0\,, \qquad \{\tilde{p}_{\Lambda[n}, U_{ab]}{}^k\} \approx 0\,, \\ &\{\tilde{p}_{\Lambda n}, U_{m\alpha}{}^\beta\} - \{\tilde{p}_{\Lambda m}, U_{n\alpha}{}^\beta\} - U_{nm}{}^k U_{k\alpha}{}^\beta \approx 0\,, \\ &\{\tilde{p}_{\Lambda n}, U_{m\alpha}\} - \{\tilde{p}_{\Lambda m}, U_{n\alpha}\} - U_{nm}{}^k U_{k\alpha} \approx 0\,.\end{aligned} \tag{4.9}$$

Similar equations hold for complex conjugates. Here the square bracket stands for a complete antisymmetrization of indices and a hat over an index means that it is not affected by the antisymmetrization.

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It is instructive then to give an explicit form of the terms quadratic in the structure functions which enter eq. (4.8)

$$
\begin{aligned}
U_{m\alpha}{}^{\beta}U_{n\beta}{}^{\gamma} - U_{n\alpha}{}^{\beta}U_{m\beta}{}^{\gamma} = \Big\{ & \frac{1}{\Delta}(\Lambda_n p_m - \Lambda_m p_n)\big(\Lambda_l \sigma^l \tilde{\sigma}^k p_k\big)_{\alpha}{}^{\gamma} + \frac{1}{\Delta}(\Lambda_n p_m - \Lambda_m p_n)\delta_{\alpha}{}^{\gamma} + \\
& + \frac{1}{\Delta}\Lambda_m \left(\sigma_n \tilde{\sigma}^k p_k\right)_{\alpha}{}^{\gamma} - \frac{1}{\Delta}\Lambda_n \left(\sigma_m \tilde{\sigma}^k p_k\right)_{\alpha}{}^{\gamma} - \\
& - \frac{1}{\Delta}\left(\Lambda_m p^2 - p_m\right)\left(\sigma_n \tilde{\sigma}^k \Lambda_k\right)_{\alpha}{}^{\gamma} + \\
& + \frac{1}{\Delta}\left(\Lambda_n p^2 - p_n\right)\left(\sigma_m \tilde{\sigma}^k \Lambda_k\right)_{\alpha}{}^{\gamma} + \left(\sigma_{nm}\right)_{\alpha}{}^{\beta} \Big\} p^2 \\
\equiv \Pi_{mn\alpha}{}^{\gamma} p^2 \,, & \\
\Pi_{mn\alpha}{}^{\gamma}\Lambda^n \approx 0 \,, \qquad \Pi_{mn\alpha}{}^{\gamma} p^n \approx 0 \,. &
\end{aligned}
\tag{4.10}
$$

Being factor of p^2 this suggests a further amendment to the $\Omega_{\min}$

$$
\begin{aligned}
& \bar{\mathcal{P}}U_{n\alpha}C^{\alpha}C^n + \bar{\mathcal{P}}U_{n\dot{\alpha}}C^{\dot{\alpha}}C^n + \frac{1}{2}\bar{\mathcal{P}}U_{nm}C^m C^n - \\
& - \frac{1}{2}\bar{\mathcal{P}}\bar{\mathcal{P}}_{\alpha}\Pi_{nm\beta}{}^{\alpha}C^m C^n C^{\beta} - \frac{1}{2}\bar{\mathcal{P}}\bar{\mathcal{P}}_{\dot{\alpha}}\Pi_{nm\dot{\beta}}{}^{\dot{\alpha}}C^m C^n C^{\dot{\beta}} \,,
\end{aligned}
\tag{4.11}
$$

where $\Pi_{nm\dot{\beta}}{}^{\dot{\alpha}}$ is the complex conjugate of $\Pi_{nm\beta}{}^{\alpha}$.

Beautifully enough, by making use of the next portion of the Jacobi identities

$$
\begin{aligned}
\{\tilde{p}_{\Lambda[a}, U_{bc]}\} + U_{[a\hat{d}}U_{bc]}{}^{d} &\approx 0 \,, \\
\Pi_{mn\alpha}{}^{\gamma}\Phi_{\gamma} + U_{m\alpha}{}^{\beta}U_{n\beta} - U_{n\alpha}{}^{\beta}U_{m\beta} + \{\Phi_{\alpha}, U_{nm}\} &\approx 0 \,, \\
\{\tilde{p}_{\Lambda[a}, \Pi_{mn]\beta}{}^{\alpha}\} + \Pi_{[a\hat{d}\beta}{}^{\alpha}U_{mn]}{}^{d} &\approx 0 \,, \\
\Pi_{[mn\delta}{}^{\gamma}U_{a]\gamma}{}^{\alpha} - U_{[a\delta}{}^{\gamma}\Pi_{mn]\gamma}{}^{\alpha} &\approx 0 \,,
\end{aligned}
\tag{4.12}
$$

and their complex conjugates, one can verify the nilpotency of our ansatz, the BRST charge in the minimal ghost sector being of the form

$$
\begin{aligned}
\Omega_{\min} = {} & \Phi_{\alpha}C^{\alpha} + \bar{\Phi}_{\dot{\alpha}}C^{\dot{\alpha}} + \tilde{p}_{\Lambda n}C^n + p^2 C + \bar{\mathcal{P}}_n\Lambda^n C^1 + \bar{\mathcal{P}}_n p^n C^2 + \\
& + \frac{1}{2}\bar{\mathcal{P}}_k\tilde{U}^k_{nm}C^m C^n + \bar{\mathcal{P}}_{\alpha}U_{n\beta}{}^{\alpha}C^{\beta}C^n + \bar{\mathcal{P}}_{\dot{\alpha}}U_{n\dot{\beta}}{}^{\dot{\alpha}}C^{\dot{\beta}}C^n + \\
& + \bar{\mathcal{P}}U_{n\alpha}C^{\alpha}C^n + \bar{\mathcal{P}}U_{n\dot{\alpha}}C^{\dot{\alpha}}C^n + \frac{1}{2}\bar{\mathcal{P}}U_{nm}C^m C^n - \\
& - \frac{1}{2}\bar{\mathcal{P}}\bar{\mathcal{P}}_{\alpha}\Pi_{nm\beta}{}^{\alpha}C^m C^n C^{\beta} - \frac{1}{2}\bar{\mathcal{P}}\bar{\mathcal{P}}_{\dot{\alpha}}\Pi_{nm\dot{\beta}}{}^{\dot{\alpha}}C^m C^n C^{\dot{\beta}} \,.
\end{aligned}
\tag{4.13}
$$

For this to be real, one has to impose the following conjugation properties on the ghost variables

$$
\begin{aligned}
& (C^{\alpha})^* = C^{\dot{\alpha}} \,, && (C^n)^* = C^n \,, && (C)^* = C \,, && (C^{1,2})^* = -C^{1,2} \,, \\
& (\bar{\mathcal{P}}_{\alpha})^* = \bar{\mathcal{P}}_{\dot{\alpha}} \,, && (\bar{\mathcal{P}}_n)^* = -\bar{\mathcal{P}}_n \,, && (\bar{\mathcal{P}})^* = -\bar{\mathcal{P}} \,, && (\bar{\mathcal{P}}^{1,2})^* = -\bar{\mathcal{P}}^{1,2} \,.
\end{aligned}
\tag{4.14}
$$

Thus, within the framework of the BRST quantization the modified formulation proves to be a theory of rank two. Our result here correlates well with that obtained previously in the alternative harmonic superspace approach [16]. Worth noting also is that a naive limit of the expression obtained to the original phase space breaks manifest Lorenz covariance ($\Lambda^i = 0$, $\Lambda^- = 0$, $\Lambda^+ = -1/p^-$), as it should.

Finally, we observe that the boundary condition which has to be imposed on the unitarizing hamiltonian (a proper hamiltonian treatment requires secondary constraints to be added to the initial hamiltonian with the corresponding Lagrange multipliers)

$$H|_{C=\bar{\mathcal{P}}=0} = H_0 = 0\,, \tag{4.15}$$

automatically satisfies the needed equation

$$\{H, \Omega_{\min}\} \approx 0\,. \tag{4.16}$$

Hence, no ghost corrections to eq. (4.15) are to be added, the latter fits to describe the unitarizing hamiltonian for the case at hand (see also ref. [24]).

5. Extension to the non-minimal ghost sector. Transition amplitude.

Having constructed $\Omega_{\min}$ and H, an extension to the non-minimal ghost sector is straightforward [18]. The irreducible constraints Φ_α, $\bar{\Phi}_{\dot\alpha}$, p^2 can be treated in the usual way. One introduces three canonical pairs of new ghost variables $(\mathcal{P}^\alpha, \bar{C}_\alpha)$, $(\mathcal{P}^{\dot\alpha}, \bar{C}_{\dot\alpha})$, $(\mathcal{P}, \bar{C})$ along with the corresponding Lagrange multipliers $(\lambda^\alpha, \pi_\alpha)$, $(\lambda^{\dot\alpha}, \pi_{\dot\alpha})$, (λ, π) (the statistics and the ghost number of the new variables are given below in the tables 1, 2 and 3). Associated with the reducible constraints $\tilde{p}_\Lambda$ are the primary ghosts and Lagrange multipliers $(\mathcal{P}^n, \bar{C}_n)$, (λ^n, π_n), as well as the secondary ones [18] $(\mathcal{P}^1, \bar{C}^1)$, $(\mathcal{P}^2, \bar{C}^2)$, (λ^1, π^1), (λ^2, π^2). A direct inspection of the structure of the ghost sector (with the use of the table 1, 2 and 3) shows the disbalance between the number of unphysical degrees of freedom and that of the ghosts introduced. This can be improved by introducing further "extra" ghosts [18]. In our case these are exhausted by $({\mathcal{P}_{(1)}}^1, \bar{C}^1_{(1)})$, $({\mathcal{P}_{(1)}}^2, \bar{C}^2_{(1)})$, $({\lambda_{(1)}}^1, {\pi_{(1)}}^1)$, $({\lambda_{(1)}}^2, {\pi_{(1)}}^2)$. The statistics and the ghost number of the new variables are gathered in the following tables 1, 2 and 3

	$\mathcal{P}^\alpha$	$\bar{C}_\alpha$	$\mathcal{P}^{\dot\alpha}$	$\bar{C}_{\dot\alpha}$	$\mathcal{P}$	$\bar{C}$	$\mathcal{P}^n$	$\bar{C}_n$	$\mathcal{P}^1$	$\bar{C}^1$	$\mathcal{P}^2$	$\bar{C}^2$
ϵ	0	0	0	0	1	1	1	1	0	0	0	0
gh	1	-1	1	-1	1	-1	1	-1	2	-2	2	-2

Table 1: Ghosts (non-minimal sector).

	λ^α	π_α	$\lambda^{\dot\alpha}$	$\pi_{\dot\alpha}$	λ	π	λ^n	π_n	λ^1	π^1	λ^2	π^2
ϵ	1	1	1	1	0	0	0	0	1	1	1	1
gh	0	0	0	0	0	0	0	0	1	-1	1	-1

Table 2: Lagrange multipliers.

	${\mathcal{P}_{(1)}}^1$	$\bar C^1_{(1)}$	${\mathcal{P}_{(1)}}^2$	$\bar C^2_{(1)}$	${\lambda_{(1)}}^1$	${\pi_{(1)}}^1$	${\lambda_{(1)}}^2$	${\pi_{(1)}}^2$
ϵ	1	1	1	1	0	0	0	0
gh	1	-1	1	-1	0	0	0	0

Table 3: Extra ghosts.

A continuation of $\Omega_{\rm min}$ to the complete relativistic phase space is now easy to perform

$$\Omega = \Omega_{\rm min} + \pi_\alpha \mathcal{P}^\alpha + \pi_{\dot\alpha}\mathcal{P}^{\dot\alpha} + \pi\mathcal{P} + \pi_n\mathcal{P}^n + \pi^1\mathcal{P}^1 + \pi^2\mathcal{P}^2 + \\ + {\pi_{(1)}}^1{\mathcal{P}_{(1)}}^1 + {\pi_{(1)}}^2{\mathcal{P}_{(1)}}^2 \,. \qquad (5.1)$$

This supplies us with the last tool needed for quantizing the theory, the corresponding transition amplitude is given by the formal path integral[9]

$$Z_\Psi = \frac{1}{2}\int D\mu\delta(\Lambda^2)\delta(1-\Lambda p)\delta(p_\Lambda p)\delta(p_\Lambda\Lambda)\delta(\Psi_\alpha)\delta(\bar\Psi_{\dot\alpha})e^{\frac{i}{\hbar}S}\,. \qquad (5.2)$$

Here the effective quantum action has the form

$$S = \int d\tau\Big(p_n\dot x^n + p_{\theta\alpha}\dot\theta^\alpha + p_{\bar\theta}{}^{\dot\alpha}\dot{\bar\theta}_{\dot\alpha} + p_{\Lambda n}\dot\Lambda^n + p_{\chi\alpha}\dot\chi^\alpha + p_{\bar\chi}{}^{\dot\alpha}\dot{\bar\chi}_{\dot\alpha} + \pi_\alpha\dot\lambda^\alpha + \pi^{\dot\alpha}\dot\lambda_{\dot\alpha} + \\ + \pi\dot\lambda + \pi_n\dot\lambda^n + \pi^1\dot\lambda^1 + \pi^2\dot\lambda^2 + {\pi_{(1)}}^1\dot\lambda^1_{(1)} + {\pi_{(1)}}^2\dot\lambda^2_{(1)} + \bar{\mathcal{P}}_\alpha\dot C^\alpha + \bar{\mathcal{P}}_{\dot\alpha}\dot C^{\dot\alpha} + \\ + \bar{\mathcal{P}}\dot C + \bar{\mathcal{P}}_n\dot C^n + \bar{\mathcal{P}}^1\dot C^1 + \bar{\mathcal{P}}^2\dot C^2 + \bar C_\alpha\dot{\mathcal{P}}^\alpha + \bar C_{\dot\alpha}\dot{\mathcal{P}}^{\dot\alpha} + \bar C\dot{\mathcal{P}} + \bar C_n\dot{\mathcal{P}}^n + \\ + \bar C^1\dot{\mathcal{P}}^1 + \bar C^2\dot{\mathcal{P}}^2 + \bar C^1_{(1)}\dot{\mathcal{P}}^1_{(1)} + \bar C^2_{(1)}\dot{\mathcal{P}}^2_{(1)} - \{\Psi,\Omega\}_D\Big)\,, \qquad (5.3)$$

with $D\mu$ being the usual Liouville measure over the full phase space and Ψ denoting the gauge fixing fermion ($\epsilon(\Psi)=1, gh(\Psi)=-1$). Given a specific form of the latter, a number of ghost (and Lagrange multiplier) integrations can be performed explicitly. For example, we can integrate out the whole bunch of variables $(C^{1,2},\bar{\mathcal{P}}^{1,2})$, $(\mathcal{P}^{1,2},\bar C^{1,2})$, $(\lambda^{1,2},\pi^{1,2})$, $({\mathcal{P}_{(1)}}^{1,2},\bar C^{1,2}_{(1)})$, $({\lambda_{(1)}}^{1,2},{\pi_{(1)}}^{1,2})$ by taking the following representation for Ψ

$$\Psi = \lambda^1\bar{\mathcal{P}}^1 + \lambda^2\bar{\mathcal{P}}^2 + \frac{1}{\epsilon}C_np^n\bar C^1 + \frac{1}{\epsilon}C_n\Lambda^n\bar C^2 + \frac{1}{\epsilon}{\lambda_{(1)}}^1\bar C^1_{(1)} + \frac{1}{\epsilon}{\lambda_{(1)}}^2\bar C^2_{(1)} + \Psi'\,. \qquad (5.4)$$

Here Ψ' does not depend on the set above and ϵ is a constant. An explicit integration which appeals to a passage to a discrete lattice attaches then four new factors to the

[9]As usual, the fermionic δ-function is defined as $\delta(\theta)=\theta$. Hence, $\delta(\Psi_\alpha)\sim\Psi_1\Psi_2\sim\Psi^2$.

path integral measure

$$Z_\Psi = \frac{1}{2}\int D\mu^{'}\delta(\Lambda^2)\delta(1-\Lambda p)\delta(p_\Lambda p)\delta(p_\Lambda\Lambda)\delta(\Psi_\alpha)\delta(\bar\Psi_{\dot\alpha})\times$$
$$\times\,\delta(C_n p^n)\delta(C_n\Lambda^n)\delta(\mathcal{P}_n p^n)\delta(\mathcal{P}_n\Lambda^n)e^{\frac{i}{\hbar}S}\,, \qquad (5.5)$$

and reduces the effective action to the relatively simple form

$$S = \int d\tau\Big(p_n\dot x^n + p_{\theta\alpha}\dot\theta^\alpha + p_{\bar\theta}{}^{\dot\alpha}\dot{\bar\theta}_{\dot\alpha} + p_{\Lambda n}\dot\Lambda^n + p_{\chi\alpha}\dot\chi^\alpha + p_{\bar\chi}{}^{\dot\alpha}\dot{\bar\chi}_{\dot\alpha} + \pi_\alpha\dot\lambda^\alpha + \pi^{\dot\alpha}\dot\lambda_{\dot\alpha} +$$
$$+\,\pi\dot\lambda + \pi_n\dot\lambda^n + \bar{\mathcal{P}}_\alpha\dot C^\alpha + \bar{\mathcal{P}}_{\dot\alpha}\dot C^{\dot\alpha} + \bar{\mathcal{P}}\dot C + \bar{\mathcal{P}}_n\dot C^n + \bar C_\alpha\dot{\mathcal{P}}^\alpha + \bar C_{\dot\alpha}\dot{\mathcal{P}}^{\dot\alpha} +$$
$$+\,\bar C\dot{\mathcal{P}} + \bar C_n\dot{\mathcal{P}}^n - \{\Psi^{'},\Omega^{'}\}_D\Big)\,, \qquad (5.6)$$

where $\Omega^{'}$ is given by eq. (5.1) with the terms involving $C^{1,2}, \pi^{1,2}, {\pi_{(1)}}^{1,2}$ omitted. In the course of the integration the standard change of variables (with unit jacobian)

$$\pi^{1,2} \longrightarrow \epsilon\pi^{1,2}\,, \qquad \bar C^{1,2} \longrightarrow \epsilon\bar C^{1,2}\,, \qquad (5.7)$$

followed by the limit $\epsilon\to 0$ has been used. Note that we do not see the compensating ghosts $(C^{1,2},\bar{\mathcal{P}}^{1,2})$ any more. The overcounting intrinsic to the sector $(C^n,\bar{\mathcal{P}}_n)$ is regulated now by the measure in eq. (5.5).

In general, we can proceed on this way. However, this seems to break manifest Lorentz covariance. In particular we found that the following ansatz for $\Psi^{'}$:

$$\Psi^{'} = \frac{1}{\epsilon}\lambda^+\bar C^- + \frac{1}{\epsilon}\lambda^-\bar C^+ + \frac{1}{\epsilon}\Lambda_i\bar C^i + \frac{1}{\epsilon}\bar C_0\bar\chi^{\dot 1} + \frac{1}{\epsilon}\bar C_{\dot 0}\chi^1 + \bar{\mathcal{P}}^i\lambda_i +$$
$$+\left(\bar{\mathcal{P}}_0 - \frac{p^1-ip^2}{\sqrt{2}p^-}\bar{\mathcal{P}}_1\right)\lambda^0 + \left(\bar{\mathcal{P}}_{\dot 0} - \frac{p^1+ip^2}{\sqrt{2}p^-}\bar{\mathcal{P}}_{\dot 1}\right)\lambda^{\dot 0} + \Psi^{''}\,, \qquad (5.8)$$

where $\Psi^{''}$ depends on $(p^n,\theta^\alpha,p_{\theta\alpha},\bar\theta^{\dot\alpha},p_{\bar\theta\dot\alpha})$ only and we switch to the light-cone notation $\lambda^n\to(\lambda^\pm,\lambda^i)$ (for fermions we write the indices explicitly $\bar C_\alpha = (\bar C_0,\bar C_1)$), reduces the integral to the standard path integral constructed with respect to the irreducible (noncovariant) subset of the Siegel constraints (2.1)

$$p_{\theta 0} - \frac{p^1-ip^2}{\sqrt{2}p^-}p_{\theta 1} = 0\,, \qquad p_{\bar\theta\dot 0} - \frac{p^1+ip^2}{\sqrt{2}p^-}p_{\bar\theta\dot 1} = 0\,, \qquad p^2 = 0\,. \qquad (5.9)$$

The explicit form of the latter is (we denote collectively $z = (x^n,\theta^1,\bar\theta^{\dot 1})$)

$$K(z_f,t_f;z_i,t_i) = \delta\left({\theta^1}_f - {\theta^1}_i\right)\delta\left(\bar\theta^{\dot 1}_f - \bar\theta^{\dot 1}_i\right)K_0(x_f,t_f;x_i,t_i)\,, \qquad (5.10)$$

where $K_0(x_f,t_f;x_i,t_i)$ is the propagator of the massless spinless relativistic particle and $({\theta^1}_i,\bar\theta^{\dot 1}_i,{\theta^1}_f,\bar\theta^{\dot 1}_f)$ denote the values of the fermions at the initial and final moments of time (boundary conditions).

Thus the path integral constructed above can be viewed as a formal covariantization of the propagator (5.10) characterizing the Siegel superparticle. Beautifully enough, this can be done with a finite number of ghost variables.

6. Discussion

In this article we have studied an alternative to the harmonic superspace approach, the latter seems to be the only method for quantizing infinitely reducible first class constraints currently available. The basic advantage of the novel technique is the existence of an explicit lagrangian formulation and the validity of the standard spin-statistics relations for all the variables involved. In contrast to the harmonic superspace approach, where one first extracts linearly independent components from originally reducible constraints and then quantizes the resulting irreducible theory, the infinite reducibility of constraints is effectively canceled by that coming from the sector of auxiliary variables. Both methods, however, correlate well yielding a theory of rank two after BRST quantization.

Turning to possible further developments, one expects the treatment of the $ABCD$ model along similar lines to be a natural next step. As has been mentioned in the Introduction, however, a proof of the equivalence of the $ABCD$ superparticle to a conventional model does not treat all constraints on equal footing. In view of this fact, the stringy extension seems to be preferable. Then, as was recently marked by Berkovits [25], a naive generalization of the present scheme to the superstring case [26] faces the zero mode problem and, hence, deserves further investigation. We suspect, however, the latter point to be a technical difficulty rather than an ideological one. Another interesting point is to make use of the present approach to test an earlier quantization proposal by Kallosh [27] (see also related works [24, 28]). The infinite proliferation of ghosts has been truncated there by imposing appropriate conditions on the ghosts variables, the latter involving specific (covariant) projectors. The phase space in our method is valid for the construction of such projectors (see also ref. [24]) and the possibility to truncate the infinite ghost tower following Kallosh's approach at the very second step seems to be tempting.

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A. Light-cone frame

In this appendix we prove the equivalence of the last of eqs. (3.8) and the pair (3.10), provided other constraints from (3.8) hold. Some details related to the analysis of the constraint system in the light-cone frame are also given.

Given the vector equation

$$-2\phi\Lambda^n + \omega p^n - i\varphi\sigma^n\bar{\chi} + i\chi\sigma^n\bar{\varphi} = 0\,, \tag{A.1}$$

the multiplication by Λ_n gives

$$\omega = 0\,. \tag{A.2}$$

Hence, the second term in (A.1) can be omitted. Passing to light cone coordinates, one has

$$-2\phi\Lambda^+ - i\varphi\sigma^+\bar\chi + i\chi\sigma^+\bar\varphi = 0\,, \tag{A.3}$$
$$-2\phi\Lambda^- - i\varphi\sigma^-\bar\chi + i\chi\sigma^-\bar\varphi = 0\,, \tag{A.4}$$
$$-2\phi\Lambda^i - i\varphi\sigma^i\bar\chi + i\chi\sigma^i\bar\varphi = 0\,, \tag{A.5}$$

where the customary notation $\Lambda^\pm = \pm\frac{1}{\sqrt2}(\Lambda^0 \pm \Lambda^3)$ is used.

It is worth mentioning now that, given a light-like vector $\Lambda^2 = -2\Lambda^+\Lambda^- + \Lambda^i\Lambda^i = 0$, the equation $(\varphi\sigma^n)_{\dot\alpha}\Lambda_n = 0$ contains only half (one) linearly independent components. Actually, taking a conventional set of σ-matrices in $R^{1|3}$ (see ref. [19])

$$\begin{aligned}
\{\sigma_n, \tilde\sigma_m\} &= -2\eta_{nm}\,, & \eta_{nm} &= \mathrm{diag}(-,+,+,+)\,,\\
\sigma^+ &= -\sqrt2\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}, & \sigma^- &= -\sqrt2\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix},\\
\Lambda_n\sigma^n &= \sqrt2\begin{pmatrix}\Lambda^+ & \frac{\Lambda^1 - i\Lambda^2}{\sqrt2}\\ \frac{\Lambda^1+i\Lambda^2}{\sqrt2} & \Lambda^-\end{pmatrix}, & \Lambda_n\tilde\sigma^n &= \sqrt2\begin{pmatrix}\Lambda^- & -\frac{\Lambda^1 - i\Lambda^2}{\sqrt2}\\ -\frac{\Lambda^1+i\Lambda^2}{\sqrt2} & \Lambda^+\end{pmatrix},
\end{aligned} \tag{A.6}$$

with $\tilde\sigma^{n\dot\alpha\alpha} = \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\sigma_{n\beta\dot\beta}$, one finds

$$(\varphi\sigma^n)_{\dot\alpha}\Lambda_n = 0 \Longrightarrow \begin{cases}\varphi^0\Lambda^+ + \varphi^1\frac{(\Lambda^1+i\Lambda^2)}{\sqrt2} = 0\\ \varphi^0\frac{(\Lambda^1-i\Lambda^2)}{\sqrt2} + \varphi^1\Lambda^- = 0\,.\end{cases} \tag{A.7}$$

Multiplying the first equation in (A.7) by $\frac{(\Lambda^1-i\Lambda^2)}{\sqrt2}$ one recovers the second one, provided the standard light-cone condition

$$\Lambda^+ \neq 0 \tag{A.8}$$

is assumed.

With the use of the explicit representation of the σ-matrices chosen, the constraint system (A.3)–(A.5) simplifies to

$$-2\phi\Lambda^+ + i\sqrt2\varphi^1\bar\chi^{\dot1} - i\sqrt2\chi^1\bar\varphi^{\dot1} = 0\,, \tag{A.9}$$
$$-2\phi\Lambda^- + i\sqrt2\varphi^0\bar\chi^{\dot0} - i\sqrt2\chi^0\bar\varphi^{\dot0} = 0\,, \tag{A.10}$$
$$-\phi(\Lambda^1 + i\Lambda^2) - i\varphi^0\bar\chi^{\dot1} + i\chi^0\bar\varphi^{\dot1} = 0\,, \tag{A.11}$$
$$-\phi(\Lambda^1 - i\Lambda^2) - i\varphi^1\bar\chi^{\dot0} + i\chi^1\bar\varphi^{\dot0} = 0\,. \tag{A.12}$$

On account of eq. (A.7) (the same holds for χ and complex conjugates), the last three equations follow from (A.9). Thus, there appears to be only one linearly independent component entering the original vector equation. The latter can be put into a covariant (scalar) form. Actually, applying the same light-cone technique to the equation

$$-2\phi - i\varphi\sigma^n\bar{\chi}p_n + i\chi\sigma^n\bar{\varphi}p_n = 0\,, \tag{A.13}$$

one recovers precisely eq. (A.9).

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